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AN EXPERIMENTAL INVESTIGATION OF NATURAL CONVECTION WAKES ABOVE A LINE HEAT SOURCE

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Abstract—The natural convection wake arising from a heated horizontal line source in liquids and in air is investigated in detail. The three-dimensional effects for a wire with a length to diameter ratio of 250 is observed in air and water with a schlieren system. The temperature field in the plume above wires with length–diameter ratios of 250 and 1200 in liquid silicone ($Pr = 6{\cdot}7$) is determined using a 20 cm Mach–Zehnder interferometer. Various wire heating rates are used yielding Grashof numbers, based on the vertical distance in the plume, in the range from 4×10^3 to $1{\cdot}7 \times 10^6$. Excellent agreement of the temperature distributions with theory are found for the larger length–diameter ratio wire at a Grashof number around 10^5. A regular natural swaying motion of the plume, observed at a high Grashof number, caused temperature fluctuations across the entire plume width.

The applicability of the idea of a virtual line and point source is considered in detail. Discrepancies in the level of the plume centerline temperature distribution in this and in previous experimental work are attributed to three-dimensional effects of the finite diameter, non-infinite length line sources.

NOMENCLATURE

c_p,	specific heat of fluid;
d,	wire diameter;
f,	similarity variable in expression for stream function;
Δf_0,	number of fringes to centerline;
g,	gravitational acceleration;
Gr_x	Grashof number based on x, $g\beta x^3 \Delta t/v^2$;
h,	heat transfer coefficient;
I,	variable integral ($\int_{-\infty}^{\infty} f'\phi \, d\eta$);
k,	thermal conductivity of fluid;
l,	wire length;
L,	interferometer optical path length;
N,	variable in temperature equation (7);
p,	pressure;
Pr,	Prandtl number of fluid;
q',	heat flux per unit length;
r,	index of refraction;
t,	fluid temperature;
Δt,	temperature excess, $t - t_\infty$;
Δt_0,	centerline temperature excess $t_0 - t_\infty$;
u,	x velocity component of fluid;
v,	y velocity component of fluid;
x,	vertical distance above line source;
y,	horizontal distance from plume centerline;
z,	axial direction parallel to line source;
β,	volumetric temperature coefficient of fluid;
ε,	number of fringes;
η,	similarity variable, $\frac{y}{x}(Gr_x/4)^{\frac{1}{4}}$;
λ,	wavelength of light;
μ,	viscosity of fluid;
v,	kinematic fluid viscosity;
ρ,	density of fluid;
ϕ,	temperature excess ratio, $\Delta t/\Delta t_0$;
Ψ,	stream function.

INTRODUCTION

STEADY state natural convection plumes arising from horizontal line sources of heat have been studied analytically since, at least, 1937, at which time Zeldovich [1], extending the similarity method used by Schlichting [2] to solve for the laminar flow velocity from a two-dimensional jet, arrived at expressions for the vertical velocity and temperature distributions in the wake. Since that time many authors have reduced the governing base flow equations to similarity form and have presented numerical and closed form solutions for the resulting differential equations. A complete literature survey of work done in the area of steady laminar natural convection plumes above a horizontal line source of heat may be found in a paper by Gebhart *et al.* [3].

The analytical work of the latter paper states the plume problem in terms of simplest variables, resolves apparent redundancies in previous work, and presents an optimum method of formulating this boundary value problem. The numerical solution of the similarity form of the boundary layer equations obtained by that method and described in [4], is used here as the theory to which the experimental results are compared.

All of the experimental work, to date, concerned with the wake from a line source, has been done in a gas, air, with a Prandtl number around 0·7. The two most recent experiments were performed by Brodowicz and Kierkus [5] in 1966 and Forstrom and Sparrow [6] in 1967. Brodowicz and Kierkus used suspended dust particles to measure velocities and an interferometer to determine the temperature distribution above a heated wire with a length–diameter ratio l/d of 3330, for a particular heating rate. The velocity profiles are in fair agreement with plume theory. The data, showing some scatter, lies mostly above theory. The temperature profiles do not parallel theory and the experimentally determined temperature similarity variable at the centerline may be calculated to be between 7 and 20 per cent below theoretical values. The experimentally determined heat flux in the wake above the wire was found to be in excellent agreement with the electrical heat input to the wire.

Forstrom and Sparrow used a thermocouple to measure the temperature distribution at various heat inputs and heights above a wire source. The data and results were interpreted to indicate that a virtual line source should be placed at two wire diameters below the actual wire in order to match the behaviour of the actual plume from a wire with the similarity solution for a line source. The value for the temperature similarity variable at the centerline determined from measurements may be calculated to be about 15 per cent below theory. A regular laminar swaying motion of the plume at high heating rates was inferred from the regular variation with time of the temperature at a fixed location in the plume.

The present paper describes and presents the results of a carefully controlled experiment which for the first time was done in a liquid, a silicone, having a Prandtl number around 6·7, (similar to water), using an interferometer to give an accurate record of the entire temperature field at any given instant. The light of the interferometer in no way disturbs the temperature field, as do other measuring instruments. This paper also discusses the experimental results compared with theory in light of the limitations imposed by experimental realities such as the finite diameter, non-infinite length source, topics which have been little more than mentioned in the past.

EXPERIMENT

Experimental apparatus

The data for the temperature distribution in the wake off of a horizontal heated wire was obtained using a 20 cm Mach–Zehnder interferometer, the description and adjustment of which is presented in a paper by Gebhart and Knowles [7]. A description of the test tank, compensating tank, and vibration insulation is included in a thesis by Knowles [8]. The fluid was a General Electric silicone fluid SF-96 with a nominal viscosity of 0·65 cks. The silicone properties relevant to the experimental and analytical results are listed in Table 1.

This silicone fluid was used because it has a Prandtl number close to that of water and yet has an electrical resistivity much greater than that

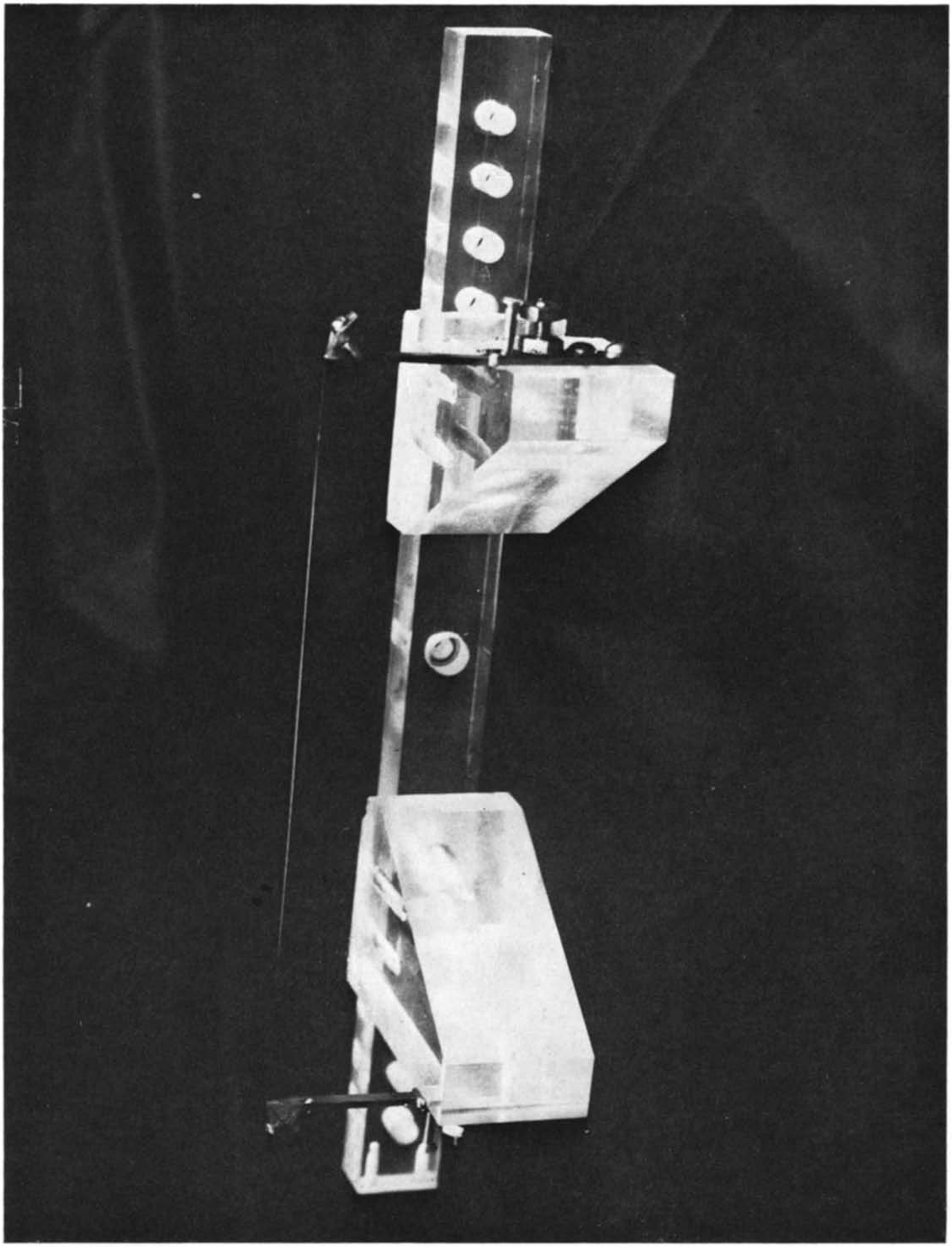


FIG. 1. Wire holder.

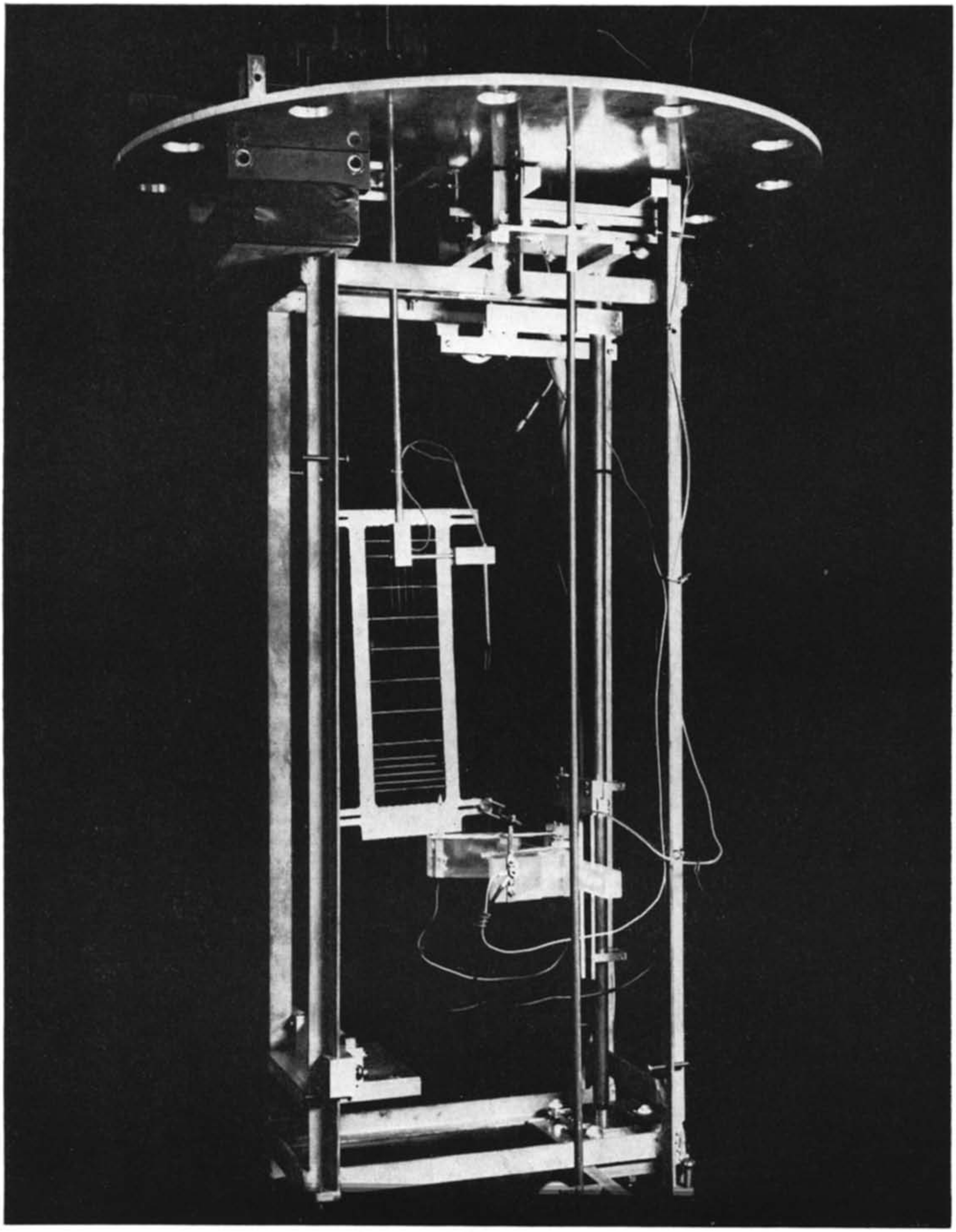
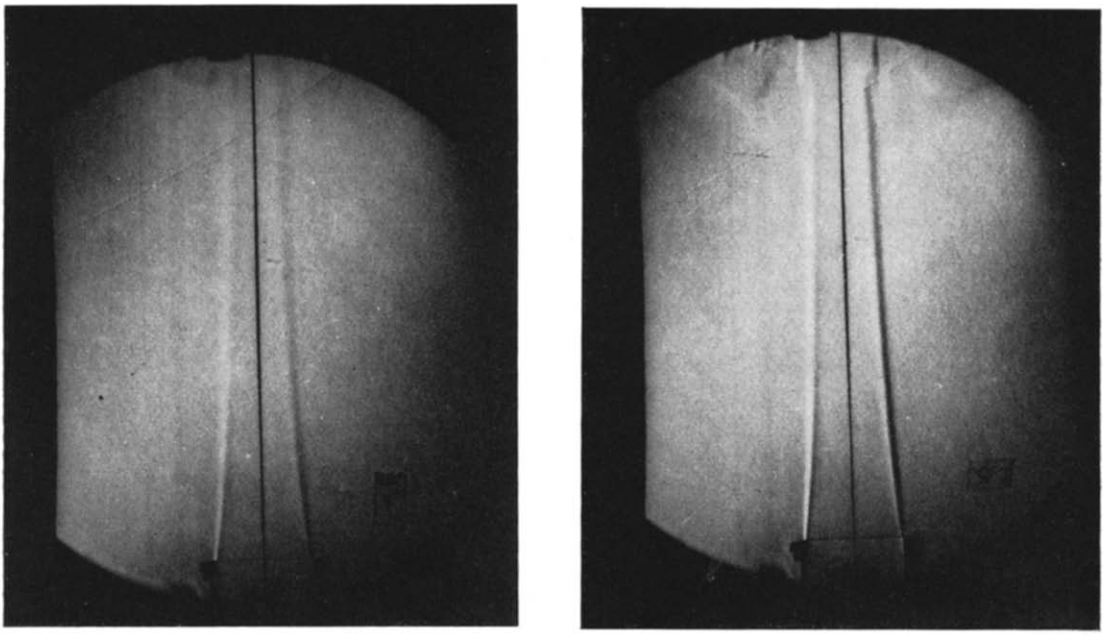
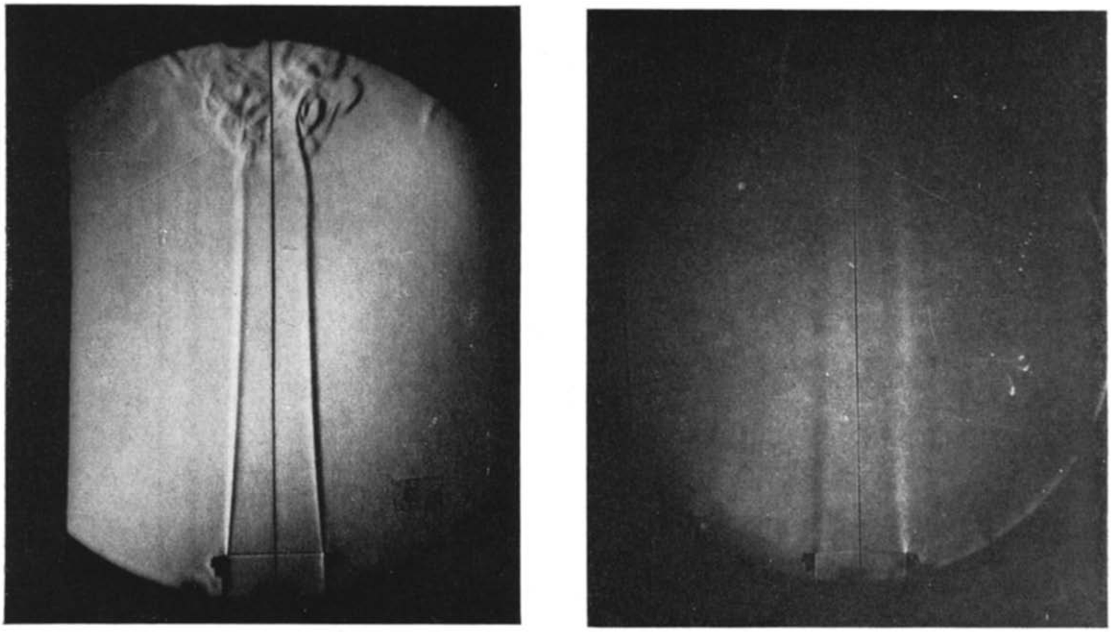


FIG. 2. View of positioning rig, wire holder and grid.



a. In water, $q'=26{\cdot}3$ Btu/hft.

b. In water, $q'=102$ Btu/hft.



c. In water, $q'=246$ Btu/hft.

d. In air, $q'=101$ Btu/hft.

FIG. 3. Schlieren photographs of end effect from 2 in. wire.

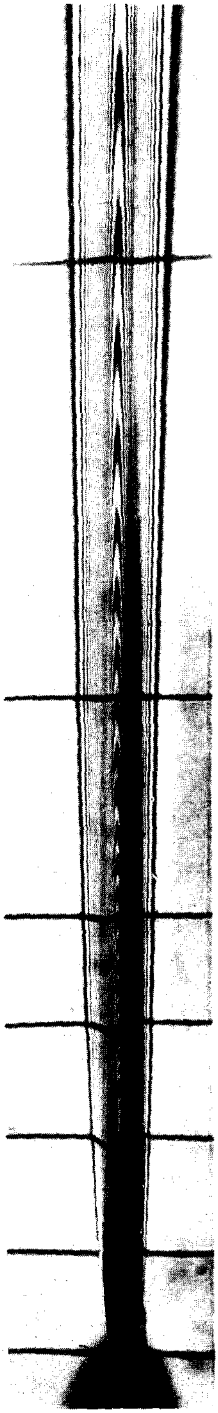


FIG. 4. Steady state interferogram of the wake from the 6 in. wire in silicone, $Pr = 6{\cdot}7$, $q' = 2{\cdot}88$ Btu/hft (bottom horizontal wire at $x = 0$ in., top horizontal wire at $x = 2\frac{1}{2}$ in.).

Table 1. Properties of G.E. silicone fluid SF-96 (0·65 cks) [9]

Viscosity (absolute, nominal)	0·65 cks
Viscosity (absolute, measured*)	0·623 cks
Refractive index	1·375
Coefficient of thermal expansion	$7{\cdot}44 \times 10^{-4}(°\text{F})^{-1}$
Thermal conductivity	0·057 Btu/hft°F
Specific heat	0·33 Btu/lb°F

* Measured by Professor F. Rodriquez of the Cornell University Department of Chemical Engineering, July, 1968.

of a standard distilled water. The high resistivity of the fluid assured no leakage of current from the wire. This particular silicone fluid also has a relatively small temperature coefficient of viscosity thus keeping this property constant across the boundary layer and for moderate changes in the ambient temperature. Due to the large change in refractive index of the silicone with respect to temperature, there are about 400 times more fringes per degree for a particular configuration than if the ambient fluid were air. The more fringes per degree permits greater accuracy for the same temperature difference or allows data to be obtained at a smaller temperature difference, which will insure the constant property assumptions. For example, the temperature difference was of the order of 1°F for these experiments.

A 0·005 in. dia. Nichrome wire, stretched between two nickel plated brass supports, Fig. 1, served as the line source. The distance between the supports was variable and 2 in. and 6 in. wire lengths were used. Good electrical contact between the wire and support was assured by the careful machining of the clamps and good mechanical contact achieved by a taut wire. The wire was kept taut in use by the deflection of the wire supports and by adjustment screws. The measured wire resistance was within 2 per cent of the calculated value.

The wire holder was supported in a positioning rig, Fig. 2, which permitted the horizontal alignment of the wire in the collimated light beam after the test tank was sealed.

Power was supplied to the wire by a Harrison Laboratories Model 865 B Power Supply. The voltage drop across the wire was measured on a Vidar 502 Electronic Integrating Digital Voltmeter. The current was determined by the voltage drop across a Leeds and Northrup 1 Ω Standard Resistance in series with the heated wire.

Two different light sources were used to obtain the data. A Stabilite Helium Neon 15 mw laser, wavelength of 6328 Å, was used for the 2 in. wire and a 100 W mercury vapor thermal source with a green interference filter passing 5461 Å light was used with the 6 in. wire. The laser, which provided more light than the thermal source, made it possible to optically enlarge the area close to the wire with the intention of distinguishing fringes down to the wire surface.

Photographic data was taken on 4 in. × 5 in. sheets of Kodak Royal Pan film with a Graflex Pacemaker Crown Graphic camera. The negatives were analyzed on a Jones and Lamson optical comparator using the 10 and 20 power magnifications.

A schlieren apparatus, consisting of 300 W zirconium light source and two 12 in. collimating mirrors, was used to determine the magnitude of the end effect (i.e. the necking in of the edges of the plume). Photographs of the schlieren image on a ground glass screen were taken at various heating rates for a 2 in. wire in water and in air. The water tank was 9 in. × 12 in. × 16 in. high with good quality glass windows and was filled with distilled water. It also served as the enclosure for tests in air.

Experimental procedure

In order to understand more fully the wake above a finite length wire, it was desirable to view the wire normal to its axis in order to obtain, at least qualitatively, the magnitude and influence of the three-dimensional end effects. The schlieren system, with the 2 in. length wire placed perpendicular to the light beam, was used for this purpose. Heating rates of 26·3, 102 and 246 Btu/hft were used to heat the wire in water. Even at the maximum sensitivity of the apparatus, heating rates lower than 26·3 Btu/hft in water did not produce density gradients large enough to yield a visible schlieren effect on the screen. In air, the lowest power setting giving a reasonable quality image and the only one used, was 101 Btu/hft, a rate causing the wire to glow. Photographic data was taken after the starting vortex from the heated wire had risen and not until the plume was well formed. The vertical direction was determined by a plumb bob suspended near the wire location.

To establish ideal test conditions for the determination of the temperature distribution required special procedures because two major difficulties were encountered in the use of liquid silicone as the ambient fluid around the line source. Because of the large number of fringes per degree, a slight temperature stratification of the silicone in the test or compensating tank, on the order of 0·002°F across the 30 in. optical path, would cause the appearance of a fringe. The tanks were insulated but large outdoor temperature variations caused a daily room temperature variation of 2–3°F. Thus a long time period of fairly constant temperature was required prior to a run in order to achieve the temperature uniformity necessary for a good infinite fringe field.

This variation in room temperature was the cause of the second difficulty. When the room temperature was rising, the temperature of the walls of the test tank was greater than the temperature of the fluid in the center of the tank. The fluid adjacent to the wall would rise, causing minor circulation with flow up the sides and down the center of the tank, directly opposed to the wake off the wire. This downflow would disturb the plume flow, causing it to twist.

The problem was partially overcome by using an air conditioner in the adjoining room to cool the surroundings thus reversing the condition for the adverse circulation. Also, initially large heating rates from the wire could be used to counteract the unfavorable circulation prior to the run. Once the circulation was minimized, test time was limited to the period before the heat added by the wire caused the tank fluid to stratify, about one hour.

It was also noted when the wake was vertical and stationary, the plume fringes would travel down and up the centerline indicating a slight twisting of the wake.

When the adverse circulation was minimized so that plume was vertical and steady, photographs of the interferograms were taken. Two different length wires, a 2 in. and a 6 in., were used in order to determine the influence of end effects on the temperature distribution. Three wire heating rates (0·72, 1·62 and 2·88 Btu/hft) were used in order to compare the similarity temperature distribution with theory over a wide range of Grashof numbers. The resulting fringe distributions were analyzed from the negatives of the pictures of the interferograms on the Jones and Lamson optical comparator. Because the low heating rates yielded low wire temperatures, radiation losses, on the order of 1 per cent, were neglected.

In order to obtain quantitative temperature information from an interferogram, the relationship between the fringe shift and temperature must be known. The relationship between the number of fringes per degree of temperature for a liquid can be shown (Knowles [7]) to be

$$\frac{\varepsilon}{\Delta t} = \frac{L\beta}{\lambda}\frac{(r^2 - 1)(r^2 + 2)}{6r} \tag{1}$$

where ε and Δt are the number of fringes and temperature difference, respectively, between two locations in the field, and r is the index of refraction.

In order to corroborate experimentally Knowles' evaluation of the number of fringes per degree a differential thermocouple system was designed to measure actual temperature differences between positions in the wake and t_∞. The results of this investigation were, however, inconclusive due to the following problems.

The maximum and minimum temperatures which could be measured to compare the interferometer results with the thermocouple results in this study are bounded by two factors. The maximum temperature difference is fixed by the maximum number of fringes (about 30) which may be resolved from an interferogram. The minimum temperature resolvable by the thermocouple system is set by the accuracy of the digital voltmeter.

The temperature differences in the wake from the 2 in. wire were sufficiently high (on the order of 1°F, 1 in. above the source) yielding about 25 fringes, but the l/d ratio was not large enough to assure that the wake flow and, consequently, the interferometer light path, was two-dimensional and of essentially a 2 in. width. For the same heating rate, a 6 in. wire produces three times as many fringes as a 2 in. wire and since a maximum of only 30 fringes could be resolved, lower heating rates for the 6 in. wire were required. The low rate in turn made the centerline temperature differences too small to be accurately determined by a single thermocouple.

The index of refraction of the silicone fluid used in this experiment was subsequently measured on a Bausch and Lomb Precision Refractometer. The results showed that even after 2 years of use the index of refraction of the sample tested was within 0·4 per cent of the manufacturers specifications. More important than the checking of the index of refraction was the determination of the change in index of refraction as a function of temperature. Results showed that $\Delta r/\Delta t = 2{\cdot}98 \times 10^{-3}/°\mathrm{F}$. The number of fringes per degree from this measurement may be determined from

$$\varepsilon/\Delta t = \frac{L}{\lambda}\frac{\Delta r}{\Delta t}. \qquad (2)$$

Table 2 lists the ratios of the number of fringes per degree, from equations (1) and (2), for the two configurations used.

The value of $\varepsilon/\Delta t$ obtained from equation (2) is within 5 per cent of the value determined from equation (1) and has been used for the temperature distributions.

Table 2. The number of fringes per degree for the two wire lengths

Wire length, L (in.)	Wave length incident light, λ (Å)	Ratio, $\varepsilon/\Delta t$ equation (1)	(2)
2	6328	25·08	23·9
6	5461	87·21	83·2

RESULTS AND DISCUSSION

End effect

Schlieren photographs taken normal to the axis of the 2 in. heated wire in water (Prandtl number of 6·7) for various heating rates, $q' = 26{\cdot}3$, 102 and 246 Btu/hft, are shown in Figs. 3a–3c. Figure 3d shows the side view for the wire in air and a heating rate of 101 Btu/hft. It can be seen that for a given length wire the necking in of the wake is greater at lower heating rates and higher Prandtl numbers. At the two higher heating rates in water, transition from laminar flow occurred at 10 in. and 8·7 in. above the wire (the fluid free surface located 12 in. above the wire), where Gr_x was 1·62 and $1{\cdot}94 \times 10^7$, as seen in Figs. 3b and 3c, respectively. The occurrence and location of this effect is thought by the writers to be due to natural disturbances present in the nominally quiescent surroundings and to the distance from the source to the free surface.

Similarity solution

The theoretical solution to which the following experimental results will be compared was obtained from the numerical solution of the governing equations, simplified by boundary layer assumptions and by the Boussinesq approximation. The y-momentum equation was neglected.

The following substitutions used by Gebhart *et al.* [3] to transform the governing equations into similarity form are presented here for reference.

$$u = \Psi_y, v = -\Psi_x \qquad (3)$$

$$\Psi = 4\nu \sqrt[4]{\left(\frac{Gr_x}{4}\right)} f(\eta) \tag{4}$$

$$\eta = \frac{y}{x} \sqrt[4]{\left(\frac{Gr_x}{4}\right)} \tag{5}$$

$$\phi(\eta) = \frac{t - t_\infty}{t_0 - t_\infty} \tag{6}$$

$$t_0 - t_\infty = N x^{-\frac{3}{5}} \tag{7}$$

where

$$Gr_x = \frac{g\beta x^3 (t_0 - t_\infty)}{\nu^2} \tag{8}$$

and

$$N = \left(\frac{(q')^4}{4^3 \mu^2 c_p^4 \rho^2 g\beta I^4}\right)^{\frac{1}{5}} \tag{9}$$

and

$$I = \int_{-\infty}^{\infty} f'\phi \mathrm{d}\eta. \tag{10}$$

The prime indicates differentiation with respect to η.

Temperature distributions

All of the data of this section was obtained from interferograms of the plume in silicone, a representative one is shown in Fig. 4 of the 6 in. wire dissipating 2·88 Btu/hft. A wire grid, seen in Fig. 4 and on the rig in Fig. 2, provides a scale for the photographs. The lowest wire is at the line source level. The following wires are located at $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1, $1\frac{1}{2}$ and $2\frac{1}{2}$ in. above the line source level. A schlieren effect due to the bending of light rays in regions of large density gradients and long optical path lengths caused the washing out of the fringes immediately around the 6 in. wire and for about the first $\frac{3}{4}$ in. of the plume.

The centerline temperature difference as a function of the distance above the line source is plotted on ln–ln coordinates, Figs. 5 and 6, for the 2 in. and 6 in. long wires, respectively. The calculated distribution is also shown, its slope being $-\frac{3}{5}$ from equation (7). The best slopes of the data, for various heating rates and wires, are listed in Table 3.

The slopes are about 7 per cent above theory for the 2 in. wire and about 5 per cent below

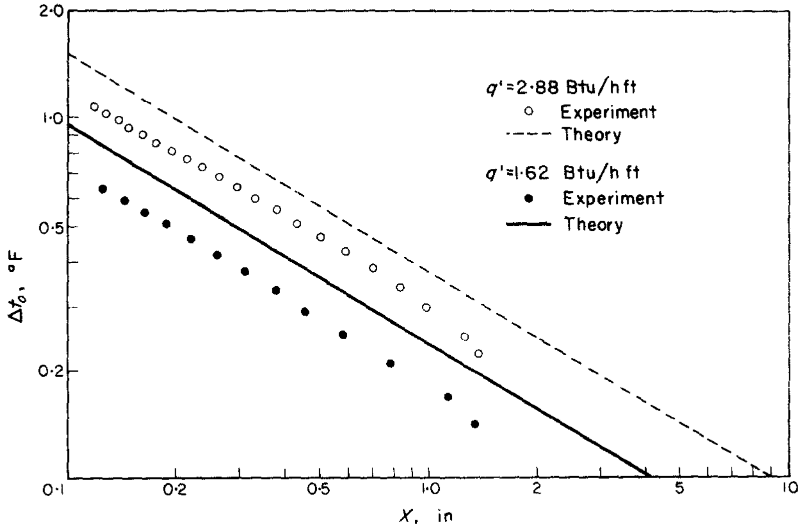


FIG. 5. Comparison of experimental results with the theoretical centerline temperature distribution, in silicone, $Pr = 6{\cdot}7$, $l = 2$ in.

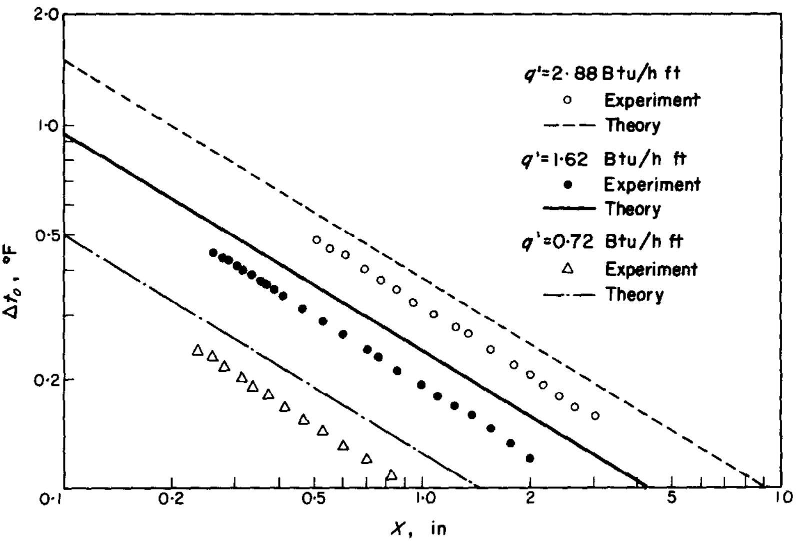


FIG. 6. Comparison of experimental results with the theoretical center line temperature distribution, in silicone, $Pr = 6{\cdot}7$, $l = 6$ in.

Table 3. Slopes of the centerline temperature distributions

	Heating rate q'(Btu/hft)	Slope n
2 in. wire	1·62	− 0·56
	2·88	− 0·56
6 in. wire	0·72	− 0·63
	1·62	− 0·635
	2·88	−0·63

theory for the 6 in. wire. The larger difference in slope from theory for the 2 in. wire is attributed to the end effects of the small l/d ratio wire, which will be discussed later.

It may be seen from Figs. 5 and 6 that while the data closely follows the theoretical slope of $-\frac{3}{5}$, the experimental curve is on the order of 20 per cent below theory for the 2 in. wire (Fig. 5) and 15 per cent below theory for the 6 in. wire (Fig. 6).

The relationship between the centerline temperature difference, heating rate, and the vertical distance may be obtained by generalizing equation (7) with respect to Gr_x and writing N as in equation (9). The following equation is obtained:

$$\frac{\Delta t_0}{q'}(4^{\frac{3}{4}}\mu c_p I) = Gr_x^{-\frac{1}{4}} \tag{11}$$

where I is a function of the Prandtl number. Figure 7 shows the theoretical curve, equation (11), and experimental points for the 6 in. wire on ln–ln coordinates. The slope of a curve through the experimental points is $-0{\cdot}26$. The level of the experimental curve is again below theory, by about 15 per cent.

Data reduced from both Forstrom and Sparrow's and Brodowicz and Kierkus' experimental results in air for wires with l/d ratios of 250 and 3330, respectively, are also plotted on Fig. 7. The slope of a curve through Forstrom and Sparrow's data points is $-0{\cdot}247$ and the level of the curve is below theory by about the same percentage as that of the present study. The curves that Forstrom and Sparrow do present in their paper are in very good agreement with theoretical predictions because of the normalizations they use in graphing their results. These normalizations obscure the disagreements with theory that are present due to end effects. The

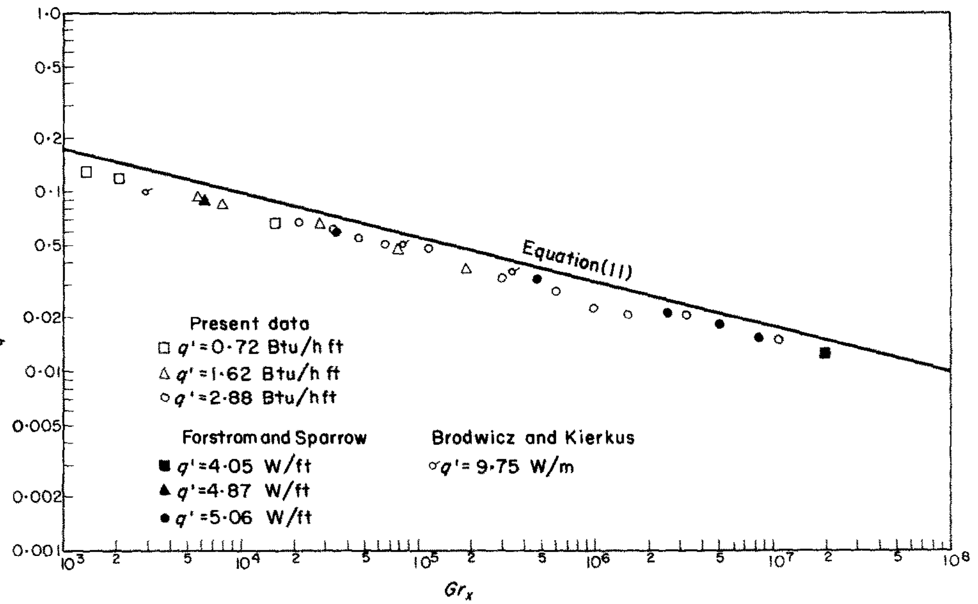


FIG. 7. Comparison of experimental results with the theoretical curve for $(\Delta t_0/q')(4^{\frac{3}{4}}\,\mu c_p l)$ as a function of Gr_x, equation (11).

slope of a line through Brodowicz and Kierkus' data is $-0{\cdot}23$ and the level again is about the same as the other data.

The consistently low data in each of the three experimental works leads one, at first, to hypothesize that end conduction losses due to the finite length wires account for appreciable heat loss through the wire to the supports. Collis and Williams [10] have experimentally shown the heat transfer coefficient from fine (d = 0·000295 cm) horizontal heated wires in air to be strongly l/d dependent for small ratios. They have postulated that the increment in the rate of heat transfer per unit length per degree is due largely to end conduction to the wire supports and possibly through the medium in an axial direction.

The fraction of heat lost due solely to end conduction through the wire to the supports has been determined analytically by equating the Joulean heat generated in a long wire to the heat lost by convection to the surrounding medium and by conduction to the wire supports, assumed to be at t_∞. The results of this calculation showed the ratio of heat lost by end conduction to the total heat generated to be

$$\frac{q_{\text{conduction}}}{q_{\text{generated}}} = \frac{kd}{hl^2}^{\frac{1}{2}} \tag{12}$$

where h, the heat transfer coefficient, was assumed constant. The value of this ratio for the 2 in. and 6 in. wire was 2·4 per cent and 0·8 per cent respectively.

The theory that most of the energy dissipated in the wire goes into the wake is further substantiated by additional experimental results of Brodowicz and Kierkus. Performing an energy balance in the wake above the wire, they found that the measured heat flux in the wake was within 1 per cent of the electrical heat input to the wire. These results imply that end conduction losses are in fact small and that the discrepancies between data and theory in Fig. 7 must be attributed to some other phenomenon.

The fact that the wires used were not infinitely long leads one to postulate three-dimensional flow with the third velocity component parallel to the wire. It is believed that this induced z-direction velocity component is of the same order of magnitude as the y-direction velocity component, at the source level. This additional velocity component, neglected in the theory, would tend to alter the velocity distribution, making both it and the temperature distribution non-similar. A thicker thermal and velocity

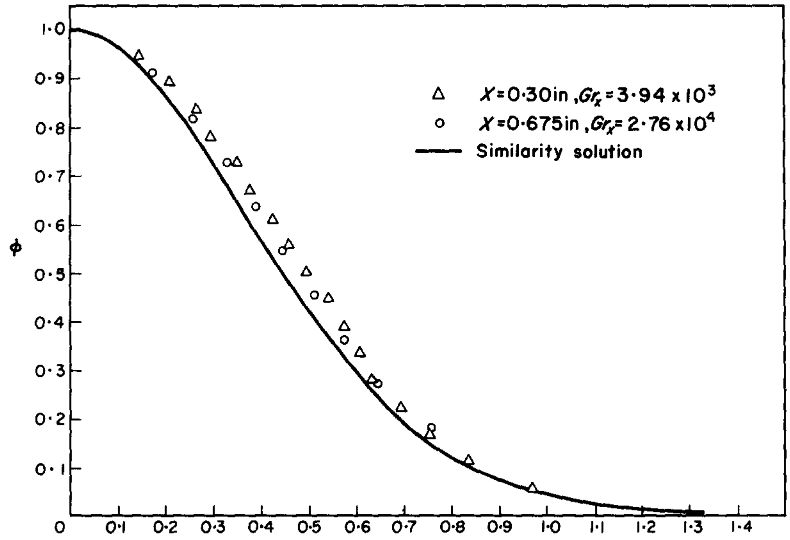


FIG. 8. Comparison of experimental results with the similarity solution temperature distribution, $l = 2$ in., $q' = 1{\cdot}62$ Btu/hft, $Pr = 6{\cdot}7$.

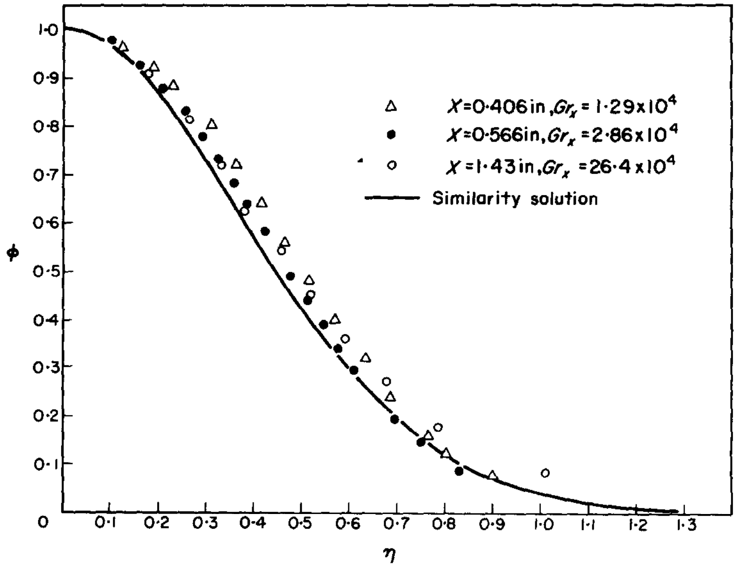


FIG. 9. Comparison of experimental results with the similarity solution temperature distribution, $l = 2$ in., $q' = 2{\cdot}88$ Btu/hft, $Pr = 6{\cdot}7$.

boundary layer possibly due to the additional velocity component could then account for the lower plume centerline temperatures.

The temperature similarity solution and the ϕ vs. η data obtained for the two wire lengths at various heating rates and Grashof numbers based on x are shown in Figs. 8–12. The temperature variable, ϕ, as defined by equation (6), is

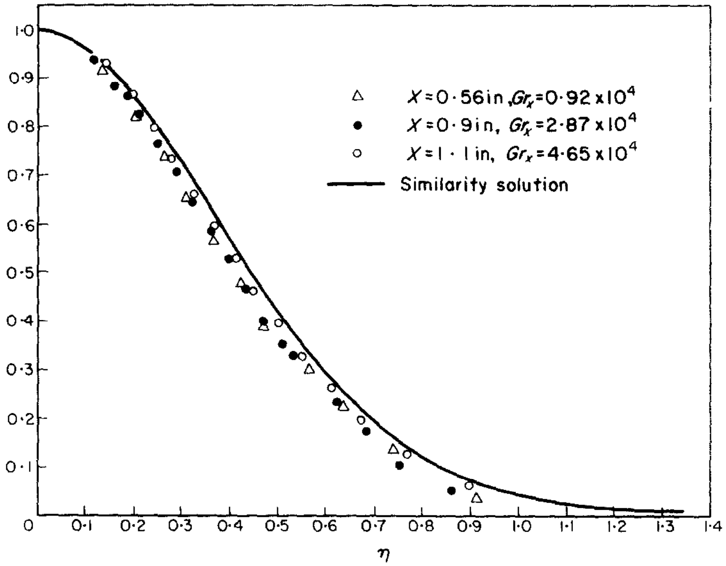


FIG. 10. Comparison of experimental results with the similarity solution temperature distribution, $l = 6$ in., $q' = 0{\cdot}72$ Btu/hft, $Pr = 6{\cdot}7$.

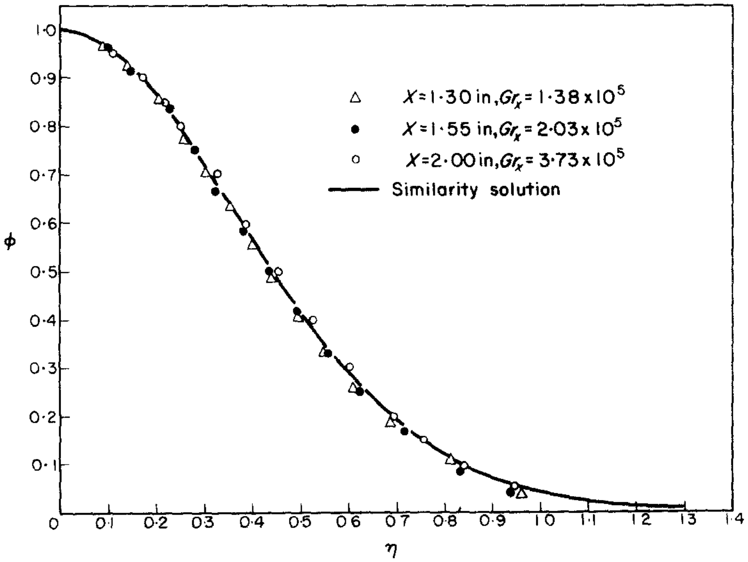


FIG. 11. Comparison of experimental results with the similarity solution temperature distribution, $l = 6$ in., $q' = 1{\cdot}62$ Btu/hft, $Pr = 6{\cdot}7$.

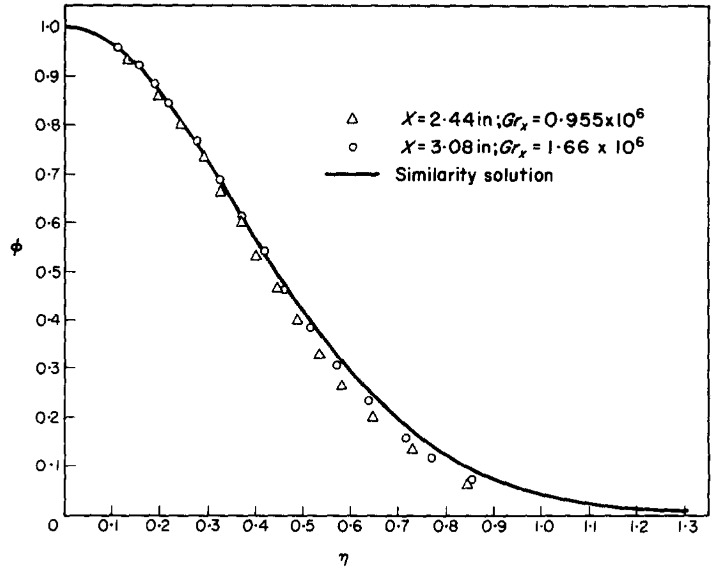


FIG. 12. Comparison of experimental results with the similarity solution temperature distribution, $l = 6$ in., $q' = 2{\cdot}88$ Btu/hft, $Pr = 6{\cdot}7$.

the ratio of two temperature differences and can therefore be written as the ratio of the number of fringes locally to those at the mid-plane. Consequently, ϕ has been evaluated directly from the interferograms. The location variable, η, as defined by equation (5), is proportional to $(q')^{\frac{1}{5}}$, and has been based on the measured electrical power input to the wire.

The ϕ vs. η points on Figs. 8 and 9, reduced from data from the 2 in. wire, for Gr_x in the range from $3{\cdot}94 \times 10^3$ to $2{\cdot}64 \times 10^5$, all lie to the right, larger η, of the theoretical curve obtained from the similarity solution of the boundary layer equations. a

The similarity temperature distributions in the plume above the 6 in. wire, Figs. 10–12, for Gr_x in the range of $0{\cdot}92 \times 10^4$–$1{\cdot}66 \times 10^6$ are in excellent agreement with theory for $q' = 1{\cdot}62$ Btu/hft, but fall slightly to the left, smaller η, for the other two heating rates.

The discrepancy in Figs. 8 and 9 may be attributed to the following. First of all, the data was evaluated at small x and low heating rates which corresponds to Gr_x not large enough to assure that the boundary layer simplifications are valid in these regions. Note that the data taken at larger x, i.e. Gr_x, more closely agree with theory. Secondly, since the l/d ratio is quite small, one cannot assume that the wake is two-dimensional or that end conduction losses are small. End conduction losses result in a lower effective heating rate for the actual plume, thus shifting the data points to the left.

A third factor whose correction would shift the experimental points into closer agreement with theory would be to measure x distances from a "virtual" source located somewhat below the actual source. Corrections in η would be larger for small x, in accordance with the spread of the experimental results. For the 6 in. wire, the data indicates that a virtual source should be placed *above* the actual wire.

The "correction" effect for the small l/d ratio wire is similar to that suggested by Forstrom and Sparrow's results. That analysis assumes a virtual source 0·083 in. (about 2 wire diameters) below the actual wire. The general reliability of this suggested correction is suspect owing to the fact that the slope of the curve (their Fig. 2) from which it was determined is 15 per cent below theory. A small change in the slope of their curve would move the x intercept and thus yield a new value for the correction.

The location of the virtual source can also be looked at from the point of view of finding a source location so that the intercept of the line $(N/\Delta t_0)^{\frac{5}{3}} = x$ passes through the origin. When the centerline temperature difference is written in terms of the number of fringes, the quantity $(\varepsilon/\Delta f_0)^{\frac{5}{3}}$ is proportional to x, the proportionality constant dependent on the heating rate. Figure 13 is a graph of the data $(1/\Delta f_0)^{\frac{5}{3}}$ vs. x for the 2 in. and 6 in. wire. The actual x intercept gives an estimate of the position a virtual source might be placed with respect to the wire source. The results of Fig. 13 are in agreement with the virtual source locations inferred from Figs. 8–12 in as much as they agree as to whether it should be above or below the actual wire.

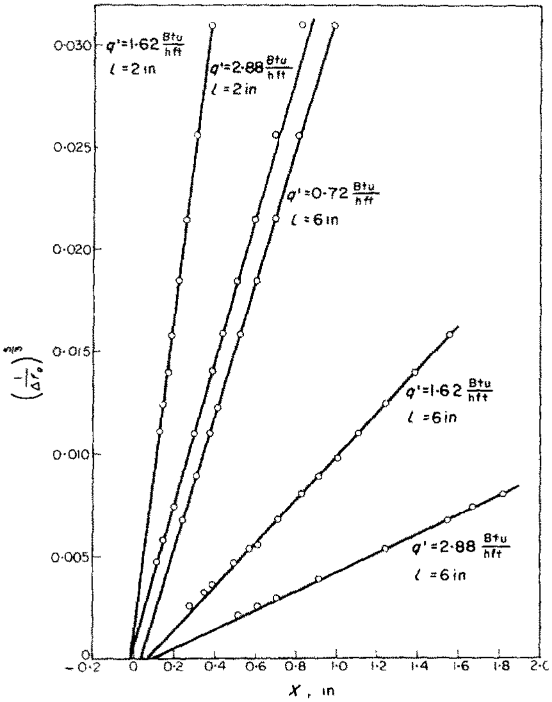


FIG. 13. Experimental results for the number of fringes to the centerline as a function of the distance above the source.

For each 2 in. wire heating rate studied, the x-axis intercept falls at x less than zero by about 0·02 in. indicating that the intercept would pass through zero if a virtual source were placed 0·02 in. (4 wire diameters) below the actual source. For the 6 in. wire, the x-intercepts range from 0·05 in. (10 wire diameters) above zero for the low heating rate to 0·100 in. (20 wire diameters) above zero for the 2·88 Btu/hft heating rate, indicating that the proper source location is above the actual source. These various intercepts show the ambiguity of the virtual source idea.

CONCLUSIONS

The analysis of the convection region arising from a horizontal line source of heat can be divided into the four regions: the limiting conduction region around the wire, the convection non-boundary layer region, the two-dimensional similarity region, and the far plume field. The separations between the regions, as in most naturally occurring phenomena, are not sharp, as are the distinguishing characteristics of each region.

The laminar two-dimensional boundary layer, or similarity region, which is the primary concern of the experimental work of this paper, can be described by the similar solution of the laminar boundary layer theory.

The near field, around the wire, can be thought of in the conduction limit, as a stagnant concentric cylinder of fluid, in which heat is transferred mainly by conduction. In this region the approximations that lead to the boundary layer equations are not valid.

The extent of this region immediately around the wire is small but, nevertheless, gives a finite flow thickness at $x = 0$. This suggests the need to use a virtual source located below the actual source. The fact the finite diameter wire does not have an infinite temperature as theory requires the temperature to be at $x = 0$ is another reason for employing an artifice, such as the virtual source.

For all experimental conditions, however, there is a flow region, immediately above the wire, to which the previous boundary layer analysis does not apply. In this convection non-boundary layer region the pressure gradients which have been neglected in the previous theory are not small and the similarity type solution is not valid. The validity of the similarity equations is limited to the region away from the origin where the flow induced pressure field and various x derivatives have been neglected. For the same Gr_x the thermal boundary layer in air is thicker than in a liquid and better approximations of the natural convection flow near the source would have to be made to determine the velocity and temperature distributions.

At large distances above the wire two regimes are possible. For the infinite length line source, the flow will remain two-dimensional and go turbulent as the Grashof number becomes large. For a finite length wire (with a small l/d) the plume goes toward an axisymmetric flow as x distances become large compared with the length of the source. This may occur before the laminar flow becomes unstable and turbulent. For the large l/d ratio wire, transition from laminar to turbulent flow is more likely to occur before the flow goes axisymmetric, the transition location being mainly Gr_x dependent.

The transition from two-dimensional toward axisymmetric flow was noticed in the data from the 2 in. wire. The variation of centerline temperature with distance above the wire as as seen on ln–ln plot Fig. 5 changes from a slope of $-0{\cdot}56$ at small x to a more negative one (less than $-0{\cdot}6$) at larger x.*

The necking down of the ends of the plume, the first step in its eventually becoming axisymmetric, can be seen in the schlieren photographs in Fig. 3. Because the wake necks in from the ends, it might be assumed that it becomes thickened in the other dimensions in order to conserve mass and energy flow. The measured thermal boundary layer thickness is in fact thicker than predicted by the two-dimensional theory, as seen in Fig. 9 at an elevation of 1·43 in. Here, at the larger elevation, the ϕ vs. η data starts, for small y, on the inside of the theoretical curve then crosses over and asymptotically approaches a larger η than does the theroretical curve, indicating a thicker thermal boundary layer.

To compare experimental results for a low l/d ratio wire with the axisymmetric point source solution, some criterion must first be established to locate the virtual point source. Two methods are suggested, only the first being adaptable to solely interferometric studies. The first technique would be to locate the point source (having the same total heat input as the line source) in x, so as to match the resulting thermal boundary layer thickness with that of the line source at the elevation above the wire where the distance between the edges of the plume is a minimum. This minimum plume width may be estimated from a schlieren picture with the light perpendicular to the wire axis (as in Fig. 3, for example). A second method, requiring that the velocity distribution be measured, would be to locate the point source so as to equate the momentum flux of the line source and the equivalent point source at some x elevation above the wire.

A regular laminar swaying motion of the wake was observed in the interferograms for the large l/d ratio wire at an elevation of 4·5 in. at a high heating rate (5 Btu/hft). This corresponds to a Gr_x of $6{\cdot}46 \times 10^6$, about the same as for the disturbances noted in water in the schlieren.

This type of boundary layer swaying motion might be termed a hydrodynamic instability and for the cases where the disturbances are amplified would likely lead to transition. It is an instability to the natural disturbances present, which are a function of the isolation of the test vessels, the nearness of a free surface, and the quiescence of the ambient surroundings.

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* For the axisymmetric plume, Δt_0 varies as x^{-1}.

Observations of the regularly swaying plume were made also by Forstrom and Sparrow in the case of air at high Grashof numbers when their thermocouple probe was placed midway between the mid-plane and edge of the thermal boundary layer. Only small fluctuations in temperature were observed when the thermocouple was placed approximately at the mid-plane. The current interferometric study showed that in a liquid the swaying motion causes substantial temperature fluctuations in time at a given x at the centerline.

No observations in silicone were made in the regime where the three-dimensional flow from the small l/d ratio wire or the two-dimensional flow from the large l/d ratio wire becomes unstable and goes turbulent.

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UNE ÉTUDE EXPÉRIMENTALE DE SILLAGES DE CONVECTION NATURELLE AU-DESSUS D'UNE SOURCE DE CHALEUR LINÉAIRE

Résumé—Le sillage de convection naturelle provenant d'une source linéaire horizontale chauffée dans des liquides et dans l'air est étudié en détail. Les effets tridimensionnels pour un fil avec un rapport longueur sur diamètre de 250 sont observés dans l'air et dans l'eau par strioscopie. Le champ de température dans le panache au-dessus de fils, avec un rapport longueur sur diamètre de 250 et de 1200, dans du silicone liquide (Pr – 6,7) est déterminé en utilisant un interféromètre de Mach–Zehnder de 20 cm. Différentes vitesses de chauffage du fil sont employées donnant des nombres de Grashof, basés sur la distance verticale dans la panache, dans la gamme de 4×10^3 à $1,7 \times 10^6$. Un excellent accord entre les distributions de température et la théorie est trouvé pour le fil de plus grand rapport longueur sur diamètre à un nombre de Grashof aux environs de 10^5. Un mouvement naturel d'oscillation régulière du panache, observé à un nombre de Grashof élevé, provoquait des fluctuations de température à travers toute la largeur du panache.

L'applicabilité de l'idée d'une source ponctuelle et linéaire virtuelle est considérée en détail. Des differences de niveau de la distribution de température sur l'axe du panache dans ce travail expérimental et dans des travaux antérieurs sont attribuées aux effets tridimensionnels des sources linéaires de longueur et de diamètre finis.

EXPERIMENTELLE UNTERSUCHUNG DES DURCH FREIE KONVEKTION HERVORGERUFENEN STRÖMUNGSFELDES ÜBER EINER LINIENFÖRMIGEN WÄRMEQUELLE

Zusammenfassung—Das durch freie Konvektion hervorgerufene Strömungsfeld, das von einer horizontalen, linienförmigen Wärmequelle in Flüssigkeiten und in Luft aufsteigt, wird im einzelnen untersucht. Die dreidimensionalen Effekte für einen Draht mit einem Verhältnis von Länge zu Durchmesser von 250 werden in Luft und Wasser mit einem Schlieren-System beobachtet. Das Temperaturfeld in der Konvektionssäule oberhalb von Drähten, für die das Verhältnis von Länge zu Durchmesser 250 bzw. 1200 beträgt, werden in flüssigem Silikon ($Pr = 6{,}7$) mit Hilfe eines 20 cm-Mach-Zehnder-Interferometers bestimmt. Verschiedene Draht-Heizleistungen dienen dazu, Grashof-Zahlen, die auf den vertikalen Abstand in der Konvektionssäule bezogen sind, im Bereich von $4 \cdot 10^3$ bis $1{,}7 \cdot 10^6$ zu erzielen. Für den Draht mit dem grösseren Verhältnis von Länge zu Durchmesser stimmt bei Grashof-Zahlen von etwa 10^5 die Temperaturverteilung ausgezeichnet mit der Theorie überein. Eine regelmässige, freie Schwingbewegung der Konvektionssäule, die bei hohen Grashof-Zahlen beobachtet wird, ruft Temperaturschwankungen über den ganzen Querschnitt der Konvektionssäule hervor.

Die Anwendbarkeit des Prinzips gedachter linien- bzw. punktförmiger Wärmequellen wird im einzelnen betrachtet. Unterschiede im Niveau der Temperaturverteilung in der Mittellinie der Konvektionssäule in vorliegender wie in früheren experimentellen Arbeiten werden dreidimensionalen Effekten zugeschrieben, wie endlichem Durchmesser und nicht unendlicher Länge der Drähte.

ЭКСПЕРИМЕНТАЛЬНОЕ ИССЛЕДОВАНИЕ СЛЕДА, ВОЗНИКАЮЩЕГО ВСЛЕДСТВИИИ ЕСТЕСТВЕННОЙ КОНВЕКЦИИ ОТ ЛИНЕЙНОГО ИСТОЧНИКА ТЕПЛА

Аннотация—Подробно изучается след, возникающий вследствии естественной конвекции от горизонтального линейного источника тепла в жидкостях и воздухе. Трехмерные эффекты для проволоки с отношением длины к диаметру 250 наблюдались в воздухе и воде с помощью шлирен-системы. Температурное поле в струйке над проволочками с отношениями длин к диаметрам 250 и 1200 в жидком кремниих ($Pr = 6.7$) определялось с помощью 20 см интерферометра Мах-Цендера. Использовались различные степени нагрева проволоки, дающие различные числа Грасгофа, построенные по вертикальному размеру струйки, в пределах от 4×10^3 до $1{,}7 \times 10^6$. Найдено хорошее соответствие результатов по распределению температур с теорией для больших отношений длины к диаметру проволоки и числах Грасгофа около 10^5. Ячеистое движение наблюдаемое при большом большом числе Грасгофа, вызывает температурные колебания по всей ширине струйки.

Подробно рассматривается применимость приближений линейного и точечного источника. Расхождение в распределении температур с известными экспериментальными работами относится за счет трехмерных эффектов линейных источников конечной длины и конечного диаметра.